
SON, THERE ARE TWO KINDS OF PEOPLE IN THIS WORLD... THOSE WHO ARE GOOD AT MATH, THOSE WHO ARE GOOD AT ENGLISH, AND THOSE WHO AIN'T GOOD AT NEITHER...

3.4 Zeros of Polynomial Functions

We know that an *n*th-degree polynomial can have at most n real zeros.

Now, in the complex number system, this statement can be improved.

That is, in the complex number system, every *n*th polynomial function has *precisely n zeros.*

Fundamental Theorem of Algebra.
If $f(x)$ is a polynomial function of degree n, where $n > 0$, then f has at least one zero in the complex number system.

Linear Factorization Theorem.
If $f(x)$ is a polynomial function of degree n, where $n > 0$, then f has precisely n linear factors
$f(x) = an(x - c_1)(x - c_2)\ldots(x - c_n)$
where $c_1, c_2, \ldots, c_n$ are complex zeros.

$$x^3 - 64 = (x-4)(x^2+4x+16)$$

$$(x-4)(x-(-2+2i\sqrt{3}))(x-(-2-2i\sqrt{3}))$$

$$(x-4)(x+2-2i\sqrt{3})(x+2+2i\sqrt{3})$$

$$x^2+4x+4 = -16 + 4$$

$$(x+2)^2 = -12$$

$$x+2 = \pm 2i\sqrt{3}$$

$$x = -2 \pm 2i\sqrt{3}$$

$$-2+2i\sqrt{3}$$

$$-2-2i\sqrt{3}$$

a) $f(x) = x - 2$

$(x-2)$

b) $f(x) = x^2 - 6x + 9$

$(x-3)(x-3)$

c) $f(x) = x^3 + 4x$

$x(x^2+4)$

$x(x+2i)(x-2i)$

d) $f(x) = x^4 - 1$

$(x^2-1)(x^2+1)$

$(x-1)(x+1)(x+i)(x-i)$

Rational Zero Test

If a polynomial has integer coeffecients, then every rational zero of the polynomial has the form:

$$\text{zero} = \frac{p}{q}$$

where p and q have no common factors and,

p = a factor of the constant term
q = a factor of the leading coefficient

Possible rational zeros:

$$\frac{\text{factors of constant}}{\text{factors of leading coeffecient}}$$

Find the rational zeros of:

$$f(x) = x^3 + x + 1$$

Find the rational zeros of:

$f(x) = x^4 - x^3 + x^2 - 3x - 6$

$p = 6$

$q = 1$

Synthetic division by -1:

-1	1	-1	1	-3	-6
		-1	2	-3	6
	1	-2	3	-6	(0)

$(x^3 - 2x^2)(+3x - 6)$

$x^2(x-2) + 3(x-2)$

$(x-2)(x^2+3)$

possible

$\frac{p}{q} = \frac{\pm 1, 2, 3, 6}{\pm 1}$

$= \cancel{1}, 2, 3, 6, -1, -2, -3, -6$

zero

$-1, 2, \pm i\sqrt{3}$

$(x+1)(x-2)(x-i\sqrt{3})(x+i\sqrt{3})$

Find the rational zeros of:

$f(x) = 2x^3 + 3x^2 - 8x + 3$

Conjugate Pairs

$1+2i \quad , \quad 1-2i$

If f is a polynomial function with real coefficients, then whenever $a + bi$ is a zero of f, $a - bi$ is also a zero of f.

Find a fourth degree polynomial with real coefficients that has -1, -1, and 3i as zeros , -3i

$(x+1)^2 (x^2+9)$

Find all the zeros of:

$f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$

given that $1 + 3i$ is a zero

Use the given zero to find all the zeros of the function:

$$f(x) = 2x^3 + 3x^2 + 50x + 75$$

zero = 5i , $-5i \Rightarrow (x^2+25)$

$12x^3 + 8x^2 - 5x + 36$

$$\frac{p}{q} = \pm\frac{1, 2, 3, 4, 6, 9, 12, 18, 36}{1, 2, 3, 4, 6, 12}$$

possible: $\pm\ 1, 2, 3, 4, 6, 9, 12, 18, 36, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}, \frac{2}{3}, \frac{3}{4}, \frac{3}{2} \ldots\ldots$

Narrowing down the search:

-Descartes's Rule of Signs

how many real pos. + neg zero's

-Upper and Lower Bounds

Renee
Descartes

1596-1650

Cartesian
coordinate

Descartes's Rule of Signs:

1. The number of positive real zeros of a polynomial is either equal to the number of variations in sign of the polynomial or less than that number by an even integer. (2)

3 or 1 positive real zero

$$f(x) = 3x^3 - 5x^2 + 6x - 4$$

Descartes's Rule of Signs:

2. The number of negative real zeros of a polynomial is equal to the number of variations in sign of the opposite of the polynomial (f(-x)) or less than that number by an even(2) integer.

0 neg. sol.

$f(x) = 3x^3 - 5x^2 + 6x - 4$

Let f(x) be a polynomial with real coefficients and a positive leading coefficient. Suppose f(x) is divided by (x - c), using sythetic division.

Upper Bounds:

If c > 0 and each number in the last row is either positive or zero, c is an upper bound for the real zeros of f.

Lower Bounds:

If c < 0 and the numbers in the last row are alternately positive and negative (zero entries count as positive or negative), c is a lower bound for the real zeros of f.

Find all the real zeros of:

$$f(x) = 12x^3-4x^2-27x+ 9$$

HW: Pg 308

#2 ,9, 18, 19, 25, 29, 38, 47, 62, 105